MODEL FOR CALCULATING THE THERMAL CONDUCTIVITY OF SOILS WITH THEIR GENESIS TAKEN INTO ACCOUNT

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The article suggests a model in which the solid component is represented in the form of three mutually intersecting ellipsoids of revolution in a cubic cell; this ensures the variable shape of particles in the entire range of changes of porosity of the system.

Soils are the products of decay of rocks as a result of lengthy physical and chemical weathering. Soils are genetically subdivided into eluvial, diluvial, alluvial, morainic, aqueous-glacial, lacustrine, and eolian soils. During the process of length conversion the shape of the particles undergoes substantial changes. In eluvial and deluvial deposits soil particles usually have angular shape. In alluvial deposits the particles are rounded off as a result of being transported by flowing water, and the soils are therefore usually composed of particles with a round shape. Clayey particles are formed mainly by chemical weathering of mineral particles, and they have a complex configuration with uneven, often corroded (pitted) surfaces, predominant in them are particles with irregular angular shape (acicular, star-shaped, clustered, chiplike).

Therefore, when a model is worked out for calculating the thermal conductivity of soils of different genesis, the change of shape of the particles has to be taken into account over the entire range of changes of porosity: from 0 to 1; this makes it possible to examine the entire cycle of conversion of primary material.

We know of many procedures and models for calculating the thermal conductivity of granular materials; their detailed review is presented in [1-4]. Most authors regard the shape of the particles as unchanging, usually in the form of spheres, and porosity changes within a limited range, from 0.26 to 1, with their different disposition in the elementary cell [5-11]. The full range of changes of porosity ($0 \le m_2 \le 1$) is attained in models with constantly contacting particles with variable shape, e.g., in the form of paraboloids of revolution [12] or bars with or without contact necks in a model with interpenetrating ellipsoids [13-15]. Zarichnyak [2] suggested a polystructural model in which the granular material is represented by a "carcass" formed by the ordered disposition of spherical particles with their closest packing (structure of first order) and a spatial network of large cavities piercing the carcass and forming with the particle a structure of second order with interpenetrating continuous components.

This model was latter improved by Dul'nev and Zarichnyak together with Eremeev [4] by replacing the ordered disposition of particles in the carcass by stochastic disposition with an averaged element which is a stream tube through the near-contact region (on the basis of the assumption that the thermal conductivity of the particles is incomparably greater than the thermal conductivity of the component in the pores). As a result it became possible to use the model in the entire range of changes of porosity of the granular system: from 0 to 1. However, replacing the entire section of particles by a narrow near-contact cylindrical region, with commensurable values of thermal conductivity of the mineral particles and of the pore component, e.g., in frozen soils, may entail some systematic errors of the calculations, mainly toward exaggeratedly large effective thermal conductivity of the system. Taking this circumstance into account and tracing the real pattern of decay of rocks and conversion of the shape of particles in the course of the length process of diagenesis, we suggest a model in which the solid component in the cubic cell is represented by three interpenetrating ellipsoids of revolution. With this arrangement, in dependence on the ratio of the semiaxes

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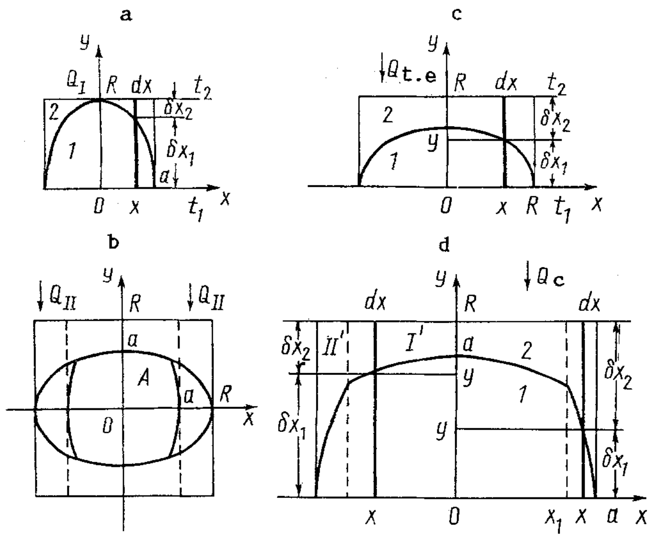

Fig. 1. Diagram for calculating Q_I (a), Q_{II} (b), $Q_{t.e}$ (c), and Q_c for $a/R \leq 1$.

of the ellipsoids a/R, the porosity of the system changes from 0 to 1, and the particles assume variegated shapes: cubic, angular, spherical, pitted, and angular cross-like; this illustrates perfectly logically the real change of shape of mineral particles in the process of genesis of deposits, i.e., the main demand on the model is fulfilled, viz., that it is an adequate picture of the real system. With the arrangement under consideration the particles always maintain contact with each other, and the system remains permanently stable and isotropic. The coordination number is constant, equal to 6, and the dependence of the thermal conductivity of the system on its porosity is effected by the change of shape of the particles with different ratios of the dimensions of the ellipsoids of revolution. In the polystructural model this dependence is connected with inconstancy of the coordination number of the particles with different close packing, which for spherical particles is physically more realistic. However, for the purpose of taking the genesis of soils into account, we specially chose the model with variable shape of the particles, but we suggest it only as one of the possible alternatives of the model representation of the structure of granular materials.

Let us consider the calculation schema of the suggested structure. Here two cases are possible: $a/R \leq 1$ and $a/R \geq 1$.

A. The Case When $a/R \leq 1$ (Fig. 1).

To calculate the heat flux in the investigated structure, we distinguish three regions: I, a cylinder with radius a in which an elongated ellipsoid of revolution with radius of the base a is inscribed; II, the region that includes the lateral spurs of two transverse ellipsoids of revolution; III, the remaining part of a cube occupied solely by the filler of the pores.

The total heat flux through the cubic cell Q is equal to the sum of the heat fluxes through the enumerated regions:

$$Q = Q_I + Q_{II} + Q_{III}. \tag{1}$$

In the calculations we adopt the following assumptions:

1. The streamlines on the interfaces between the solid component and the medium do not curve.

2. The isothermal surfaces are planes passing through the center and the upper side of the cube perpendicularly to the line of heat flow. Between these surfaces a constant temperature gradient $\Delta t = t_2 - t_1$ is specified.

Let us consider each region separately.

Region I ($0 \leq x \leq a$) (Fig. 1a). The amount of heat transmitted through an annular element with width dx is equal to

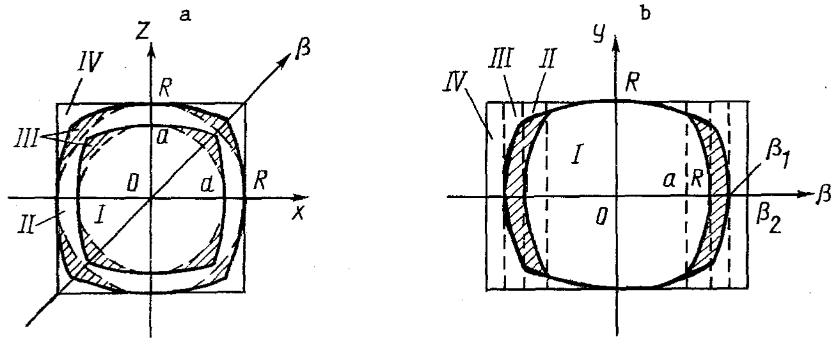

Fig. 2. Diagram of the distribution of heat fluxes of the model for $a/R \geq 1$: a) view from top; b) section along the diagonal ($\beta_1 = \sqrt{2}aR/\sqrt{a^2 + R^2}$; $\beta_2 = \sqrt{2}R$).

$$dQ_I = \frac{\Delta t}{\Omega_x} = \frac{\Delta t dS}{\dfrac{\delta x_1}{\lambda_1} + \dfrac{\delta x_2}{\lambda_2}}, \tag{2}$$

where $\Omega_x = \left(\dfrac{\delta x_1}{\lambda_1} + \dfrac{\delta x_2}{\lambda_2}\right)\dfrac{1}{dS}$ is the thermal resistance of an annular element with cross-sectional area dS.

It follows from Fig. 1a that $\delta x_1 = y$, $\delta x_2 = R - y$ and $dS = 2\pi x dx$. The correlation between y and x is found from the equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{R^2} = 1. \tag{3}$$

Hence $y = R/a\sqrt{a^2 - x^2}$.

In view of this relation we obtain

$$\Omega_x = \frac{R(a - k\sqrt{a^2 - x^2})}{2\pi\lambda_2 a x dx}, \tag{4}$$

where $k = 1 - \lambda_2/\lambda_1$.

Integrating the expression $dQ_I = \Delta t/\Omega_x$ from 0 to a, we find

$$Q_I = \frac{2\pi\lambda_2\Delta t}{R}\int_0^a \frac{xdx}{a - k\sqrt{a^2 - x^2}} = \frac{2\pi\Delta t\lambda_2 a^2}{kR}\left(\frac{1}{k}\ln\left|\frac{1}{1-k}\right| - 1\right). \tag{5}$$

Region II (Fig. 1b). To evaluate the heat flux Q_{II} we first calculate the flux $Q_{t.e}$ through the entire transverse ellipsoid of revolution. Then we subtract from it the heat flux Q_c passing through the cylinder with radius a in which the solid component is contained in the form of the barrel A.

Let us consider in Fig. 1c the section of the region in which the solid component is contained in the form of a transverse ellipsoid of revolution.

The following is the initial equation:

$$\frac{y^2}{a^2} + \frac{x^2}{R^2} = 1. \tag{6}$$

From that, taking the relation $dS = 2\pi x a dx/R$ into account, we find

$$\Omega_x = \frac{R^2 - ak\sqrt{R^2 - x^2}}{2\pi\lambda_2 a x dx}. \tag{7}$$

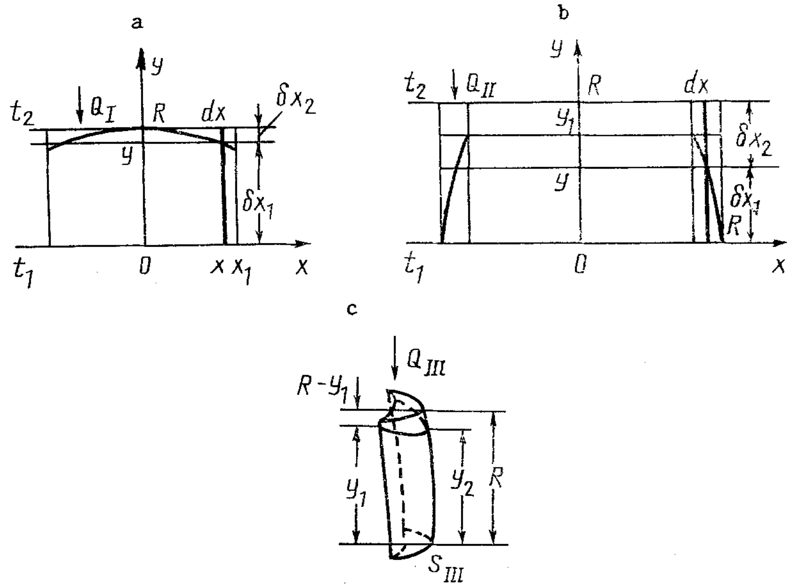

Fig. 3. Diagram for the calculation of Q_I (a), Q_{II} (b), and Q_{III} (c) for $a/R \geq 1$.

When we integrate expression (2) from 0 to R, we finally obtain the heat flux through the region with the solid component in the shape of a transverse ellipsoid of revolution:

$$Q_{t.e} = \frac{2\pi\Delta t\lambda_2 R}{k}\left(\frac{R}{ak}\ln\left|\frac{1}{1-ak/R}\right| - 1\right). \tag{8}$$

The heat flux through a cylinder with radius a containing the solid component in the shape of the letter A, formed by the intersection of a longitudinal and two transverse ellipsoids of revolution, can be represented as a system of two fluxes through a cylinder with the radius $x_1 = aR/\sqrt{a^2 + R^2}$ (region I') and a cylindrical ring with wall thickness $a - x_1$ (region II') (Fig. 1d).

In the region I' ($0 \leq x \leq x_1$) relations (6) are obeyed, and $dS = 2\pi x dx$. Therefore,

$$\Omega_x = (R^2 - ak\sqrt{R^2 - x^2})/2\pi R\lambda_2 x dx. \tag{9}$$

Integration of expression $dQ_I' = \Delta t/\Omega_x$ from 0 to x_1, with the relations written above taken into account, yields

$$Q_{I'} = \frac{2\pi\lambda_2\Delta t R^2}{ak}\left[\frac{R}{\sqrt{a^2+R^2}} - 1 + \frac{R}{ak}\ln\left|\frac{1-ak/\sqrt{a^2+R^2}}{1-ak/R}\right|\right]. \tag{10}$$

In region II' ($x_1 \leq x \leq a$) relations (3) and (4) apply.

Integrating the expression $dQ_{II'} = \Delta t/\Omega_x$ from x_1 to a, we obtain

$$Q_{II'} = \frac{2\pi\Delta t a^2\lambda_2}{kR}\left[\frac{1}{k}\ln\left|\frac{1}{1-ak/\sqrt{a^2+R^2}}\right| - \frac{a}{\sqrt{a^2+R^2}}\right]. \tag{11}$$

Thus, the heat flux through the cylindrical region with radius a is equal to

$$Q_c = Q_{I'} + Q_{II'} = \frac{2\pi\Delta t\lambda_2}{k}\left[\left(\frac{R^3}{a} - \frac{a^3}{R}\right)\Big/\sqrt{a^2+R^2} - R^2/a + \right.$$
$$\left. + \frac{R^3}{ka^2}\ln\left|\frac{1-ak/\sqrt{a^2+R^2}}{1-ak/R}\right| + \frac{a^2}{kR}\ln\left|\frac{1}{1-ak/\sqrt{a^2+R^2}}\right|\right]. \tag{12}$$

The heat flux through the region that includes the lateral spurs of one transverse ellipsoid of revolution is equal to $Q_{\ell.s} = Q_{t.e} - Q_c$.

The heat flux $Q_{\ell.s}$ relates to the area $S_{\ell.s}$ of the base of the lateral spurs of one transverse ellipsoid of revolution. When there are two mutually intersecting transverse ellipsoids of revolution, there is a common area of superposition which has to be taken into account in the calculation of the heat flux.

The area of the base of the lateral spurs of two transverse ellipsoids of revolution is divided equally by the diagonals of the square. We denote by S_g this area of one lateral ellipsoid of revolution. Then the heat flux through region II that includes the lateral spurs of two intersecting transverse ellipsoids of revolution can be expressed as $Q_{II} = 2Q_{\ell.s}S_g/S_{\ell.s}$. The area $S_{\ell.s}$ is equal to $\pi aR - \pi a^2$.

It can be shown that

$$S_g = \pi aR - \pi a^2/2 - 2aR \arcsin(a/\sqrt{a^2 + R^2}). \tag{13}$$

Taking formulas (8) and (12) into account, we obtain finally for the heat flux through region II:

$$Q_{II} = \frac{4\pi\Delta t\lambda_2 R}{k}\left\{\frac{R}{a}\left(1 - \frac{a}{R}\right)\left(1 - \frac{R}{ka}\ln\left|\frac{1}{1 - ak/R}\right|\right) - \right.$$
$$\left. - \frac{R}{a}\left(1 - \frac{a^4}{R^4}\right)\left[(1/\sqrt{1 + a^2/R^2}) + \frac{R}{ka}\ln\left|1 - \frac{ka}{R}\Big/\sqrt{1 + a^2/R^2}\right|\right]\right\} \times$$
$$\times\left[1 - \frac{4\arcsin(a/R\sqrt{1 + a^2/R^2}) - \pi a/R}{2\pi(1 - a/R)}\right]. \tag{14}$$

<u>Region III.</u> The area occupied by the filler in the base of the ellipsoids is equal to

$$S_{III} = 4R^2 - 2\pi aR + 4aR\arcsin(a/\sqrt{a^2 + R^2}). \tag{15}$$

Then

$$Q_{III} = \frac{\lambda_2\Delta t S_{III}}{R} = \frac{\lambda_2\Delta t}{R}[4R^2 - 2\pi aR + 4aR\arcsin(a/\sqrt{a^2 + R^2})]. \tag{16}$$

From the found expressions for Q_I, Q_{II}, and Q_{III} we can calculate the resulting heat flux Q by formula (1). On the other hand, it is equal to $4\lambda\Delta tR$ (here λ is the effective thermal conductivity of the entire system).

For the thermal conductivity of the system in the region $a/R \leq 1$ we finally obtain from these relations:

$$\lambda = \lambda_2\left\{\frac{\pi}{k}\left\{\frac{R}{a}\left(1 - \frac{a}{R}\right)\left(1 - \frac{R}{ak}\ln\left|\frac{1}{1 - ak/R}\right|\right) - \right.\right.$$
$$\left. - \frac{R}{a}\left(1 - \frac{a^4}{R^4}\right)\left[\frac{1}{\sqrt{1 + a^2/R^2}} + \frac{R}{ak}\ln\left|1 - \frac{ak}{R\sqrt{1 + a^2/R^2}}\right|\right]\right\} \times$$
$$\times\left[1 - \frac{4\arcsin(a/R\sqrt{1 + a^2/R^2}) - \pi a/R}{2\pi(1 - a/R)}\right] + 1 - \frac{\pi a}{2R} -$$
$$\left. - \frac{\pi a^2}{2kR^2}\left(\frac{1}{k}\ln|1 - k| + 1\right) + \frac{a}{R}\arcsin\frac{a}{R\sqrt{1 + a^2/R^2}}\right\}. \tag{17}$$

B. The Case When $a/R \geq 1$.

To calculate the heat flux, we divide the system into four regions (Fig. 2). We represent the central part of the figure in Fig. 2 (region I) by a cylinder with radius $x_1 = aR/\sqrt{a^2 + R^2}$. Region II is a cylindrical ring with inner and outer radii equal to x_1 and R, respectively. Region III consists of spurs of the solid component in the diagonal directions of the section. Region IV is filled with the medium.

Let us consider each region separately.

Region I ($0 \le x \le x_1$) (Fig. 3a). From above the solid component is bounded by part of the solid ellipsoid of revolution around the Oy-axis. At that relations (3) and (4) are fulfilled.

Integrating $dQ_I = \Delta t/\Omega_x$ from 0 to x_1, we obtain

$$Q_I = \frac{2\pi\Delta t\lambda_2 a^2}{kR}\left[\frac{1}{k}\ln\left|\frac{1-ak/\sqrt{a^2+R^2}}{1-k}\right| + \frac{a}{\sqrt{a^2+R^2}} - 1\right]. \tag{18}$$

Region II ($x_1 \le x \le R$) (Fig. 3b). The interface between the solid component and the medium is described by Eq. (6), and the thermal resistance by formula (9). We integrate the relation $dQ_{II} = \Delta t/\Omega_x$ with respect to x from x_1 to R. Then we obtain

$$Q_{II} = \frac{2\pi\Delta t\lambda_2 R^2}{ak}\left[\frac{R}{ak}\ln\left|\frac{1}{1-ak/\sqrt{a^2+R^2}}\right| - \frac{R}{\sqrt{a^2+R^2}}\right]. \tag{19}$$

Region III (Fig. 3c). It contains the angular projections of the solid component in the cube. For convenience of the calculation we rectify the sloping projections and take the height equal to $y_1 = aR/\sqrt{a^2+R^2}$, thereby the reduction of height is compensated to some extent by the slope of the rib y_2. As the area of the element we adopt the area of its base S_{III}.

Then we obtain

$$\Omega_x = \frac{1}{S_{III}}\left[\frac{y_1}{\lambda_1} + \frac{R-y_1}{\lambda_2}\right] = \frac{R(1-ak/\sqrt{a^2+R^2})}{\lambda_2 S_{III}}, \tag{20}$$

where $S_{III} = S_b - \pi R^2 = 4aR\arcsin(R/\sqrt{a^2+R^2}) - \pi R^2$; S_b is the area of the base of the solid component.

Thus:

$$Q_{III} = \frac{\lambda_2\Delta t\,[4aR\arcsin(R/\sqrt{a^2+R^2}) - \pi R^2]}{R(1-ak/\sqrt{a^2+R^2})}. \tag{21}$$

Region IV.

$$S_{IV} = 4R^2 - S_b, \qquad Q_{IV} = \frac{\lambda_2\Delta t S_{IV}}{R} = 4\lambda_2\Delta t\,[R - a\arcsin(R/\sqrt{a^2+R^2})]. \tag{22}$$

The total heat flux is equal to the sum of the fluxes Q_I, Q_{II}, Q_{III}, and Q_{IV}. On the other hand we have $Q = 4\lambda\Delta tR$. Hence we finally obtain for the effective thermal conductivity of the model:

$$\lambda = \lambda_2\left\{1 - \frac{\pi}{4} - \frac{\pi a^2}{2kR^2}\left(\frac{1}{k}\ln|1-k| + 1\right) + \frac{\pi}{2k}\left(\frac{a^2}{R^2} - \frac{R^2}{a^2}\right)\left(\frac{1}{k}\ln|1-k/\sqrt{1+R^2/a^2}| + 1/\sqrt{1+R^2/a^2}\right) + \frac{k}{(\sqrt{1+R^2/a^2}-k)}\left[\frac{a}{R}\arcsin(R/a\sqrt{1+R^2/a^2}) - \frac{\pi}{4}\right]\right\}. \tag{23}$$

In the found formulas (17) and (23) there appears the ratio of the semi-axes of the ellipsoids of revolution a/R which is a single-valued function of the porosity m_2 or of the rleative volume of the solid component m_1. The correlation between m_1 and a/R is found in the following form:

for $a/R \le 1$

$$m_1 = \frac{\pi a^2}{6R^2}\left\{1 - \frac{R}{a}\left(1 - \frac{a^2}{R^2}\right) + \frac{1-a/R}{\sqrt{1+a^2/R^2}}\left[\frac{R}{a}\left(1+\frac{a}{R}\right) - \frac{1}{1+a^2/R^2} + 3\right]\right\}, \tag{24}$$

for $a/R \ge 1$

$$m_1 = \frac{\pi a^2}{6R^2}\left[1 - \frac{1-R^2/a^2}{\sqrt{1+R^2/a^2}}\right] + \frac{2+R^2/a^2}{16(1+R^2/a^2)}\left(\frac{1}{\sqrt{1+R^2/a^2}} + \frac{1}{\sqrt{1+2R^2/a^2}}\right)\left[\frac{4a}{R}\arcsin(R/a\sqrt{1+R^2/a^2}) - \pi\right]. \tag{25}$$

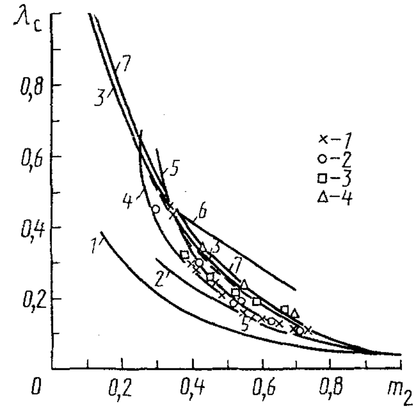

Fig. 4. Dependence of the thermal conductivity [W/(m·K)] of dry soils on porosity m_2: calculation (solid lines): 1) after Maxwell and Odelevskii [18]; 2) after Kaganer [11]; 3) by formula (30); 4) after Dul'nev and Sigalova [8]; 5) after Bogomolov [5]; 6) after Lyalikov [7]; 7) after Dul'nev and Zarichnyak [4]; experimental data: 1) after Bogomolov [5]; 2) after Chudnovskii [1]; 3) after Kammerer [16]; 4) after Kaufman [16]; dashed line) experimental curve [17].

We investigate the obtained formulas in the limit cases.

1. Let $a/R = 0$. Then $m_1 = 0$, $m_2 = 1$ and $\lambda = \lambda_2$.

2. Let $a/R \to \infty$. Then $m_1 = 1$, $m_2 = 0$ and $\lambda = \lambda_1$.

3. Let $k = 0$, i.e., $\lambda_1 = \lambda_2$. Then for any m_1 $\lambda = \lambda_1$.

4. Let $k = 1$, i.e., $\lambda_2 = 0$. Then for any m_1 $\lambda = \lambda_2 = 0$.

5. Let $a/R = 1$, i.e., we have a sphere. Then $m_1 = \pi/6$ and $\lambda = \lambda_2\{(\pi/2k)\cdot[k^{-1}\ln[1/(1 - k)] - 1] + 1 - \pi/4\}$.

It can be seen that the suggested formulas for calculating thermal conductivity operate normally on limit transitions.

Before we directly begin the calculations, we stipulate the following. Formulas (17) and (23) were obtained with division of the elementary cell by an adiabatic surface parallel to the heat flux. At the same time Dul'nev and Zarichnyak [4] proved that adiabatic division yields excessively low thermal conductivity of the system (λ_{ad}), and isothermal division yields an exaggerated value (λ_{is}). In calculations we therefore have to take the mean value of λ:$\bar{\lambda} = (\lambda_{ad} + \lambda_{is})/2$. This relation can be represented in the form $\lambda = (1 + \Delta\lambda/\lambda_{ad})\lambda_{ad}$ (here $\Delta\lambda = \bar{\lambda} - \lambda_{ad}$). Obviously, for $m_2 = 0$, $m_2 = 1$, and $\lambda_1 = \lambda_2$ we have $\Delta\lambda = 0$. The maximal differences $\Delta\lambda$ are attained in the central region of change of porosity m_2, of the order 0.2-0.5, and for $\lambda_2 = 0$ [4]. Proceeding from these considerations, we can represent the correction factor $B = \Delta\lambda/\lambda_{ad}$ in general form by the following approximate relation:

$$B = B_{max} \sin \pi m_2, \tag{26}$$

where $0 \le m_2 \le 1$; $B_{max} = (\Delta\lambda)_{max}/\lambda_{ad}$.

The parameter B_{max} depends on the ratio of the thermal conductivity of the components: $\nu = \lambda_2/\lambda_1$. The nature of this dependence is determined by the geometry of the model. For our model we recommend the following relation:

$$B_{max} = \frac{1{,}3}{1 + 0{,}56\nu - 0{,}26\nu^2} - 1, \tag{27}$$

where $0 \le \nu \le 1$.

Therefore, the final calculations of λ have to be carried out by the following formula:

$$\lambda = (1 + B_{max} \sin \pi m_2) \lambda_{ad}, \tag{28}$$

where λ_{ad} are the values of λ calculated by formulas (17) and (23).

Figure 4 presents the results of calculations of the thermal conductivity of dry soils by different formulas. For the sake of comparison the figure also shows the experimental data of [1, 5, 16, 17]. It should be noted that in consequence of the great differences between the thermophysical parameters of mineral particles and air, dry soils have the poorest conditions for comparing the theoretical formulas. In the calculations we adopted the following values of λ [W/(m·K)]: 0.025 for air and 6.0 for the mineral particles of quartz sand.

It can be seen that the formula by Odelevskii [18] (particles with cubic shape) and Maxwell (spherical particles) (curve 1) yields data that are too low; this follows from the very structure of the model in which heat transfer between is effected obligatorily through the atmospheric air.

With porosity smaller than 0.4, Kaganer's formula [11] (curve 2) begins to yield values of λ that are too low because, as Kaganer believes, with $m_2 < 0.414$ the heat flux through the pores becomes negative.

Lyalikov's formula [7] (curve 6) yields distinctly exaggerated data because the model assumes that particles move apart in one direction only, viz., horizontally, i.e., with any porosity of the system the contacts between particles are maintained.

Satisfactory agreement with the experimental data is attained in calculations by the formulas of Bogomolov [5] (curve 4), Dul'nev and Sigalova [8] (curve 5), Dul'nev and Zarichnyak [4] (curve 7), and (30) of the present work (curve 3). It should be noted that the models in [5] and [8] were constructed with the closest packing of spherical particles (tetrahedral and hexagonal disposition of the particles), and for $m_2 \approx 0.26\lambda$ it tends to infinity. In fact, Fig. 4 shows that near $m_2 = 0.26$ curves 4 and 5 begin to rise steeply.

A logically more correct dependence of λ_c on m_2 in the entire range of change of porosity of granular materials ($0 \le m_2 \le 1$) is obtained by the formulas of Dul'nev's and Zarichnyak's model [4] (curve 7) and the models suggested by us (curve 3). Both models ensure very close correspondence of the results of calculating λ_c (the differences do not exceed 5%) with which our experimental data [17] for dry soils (sand, sandy loam, and loam) and Kaufman's data [16] for fine-grained materials of brick and limestone almost coincide.

In conclusion, we note that the proposed model is also suitable for calculating the thermal conductivity of snow cover because there the pattern of the change of structure in diagenesis is similar (only in reversed order).

NOTATION

Q, heat flux through the surface area S; t, temperature; Ω, thermal reistance; λ, thermal conductivity; V, volume; m, volume content of soil components; x and y, coordinates; a and R, minor and major semi-axes of ellipsoids of revolution, respectively. Subscripts: 1, solid soil component; 2, pore filler of the soil.

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RADIATIVE-CONVECTIVE HEAT TRANSFER IN A SYSTEM OF TWO POROUS PLATES

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The solution to a system of equations is investigated, which describe the process of heat transfer between a heated gas and porous plates and which consider the convective heat transfer and the radiation of the particles in the porous plates.

A method is proposed to obtain superadiabatic temperatures in the combustion zone during the burning of a low-calorie gaseous fuel in a system of two highly porous plates, due to the recuperation of thermal energy by the skeletons of the porous plates.

The idea of recovering the heat by radiation in the working zone was suggested by Japanese scientists R. Echigo, Y. Yoshizava, et al. [1, 2]. The one-dimensional transient problem of burning gaseous fuel in porous material was solved [1] considering convective and radiative heat transfer. The combustion rate was found from the Arrhenius equation. The initial temperature profile of the gas was used for the initial condition. The effect was studied of the optical thickness, the absorption coefficient, and the position of the reaction zone relative to the porous layer boundaries on the maximum gas temperature. An original furnace design was proposed [2] for realizing a catalytic reaction, the energy for which was fed in by radiation through a screen transparent to radiation from the particles of the porous layer. The air was heated internally by combustion of an air mixture with a low-grade fuel. The screen is not permissible to material.

Here we examine the method of recuperating the heat by radiation within a system of two highly porous plates. The low-calorie gaseous fuel (diluted natural gas, paint vapors, etc.) is fed through one plate into a narrow gap, where it is burned. The combustion products filter through the second plate. It is proposed to utilize the method of radiative heat transfer to heat the porous plate, through which air is pumped to preheat it by heat transfer between the gas and the skeleton of the porous plate. Preheating the incoming gas permits temperatures above the adiabatic temperature. Only the thermal balance problem has been examined [2], without specifying the chemical and kinetic properties of the system.

A stationary process is examined. The flow of gas is assumed constant. The characteristic dimensions of the plates are much larger than the distance between them, which makes it possible to neglect edge effects and examine a one-dimensional problem. The optical characteristics are taken to be those of a system of spheres of identical radius R_i and emissivity ε_i [3]. The gas flowing through the plates is assumed to be optically transparent. Thermal conduction processes are neglected through the gas and between the particles of the porous plates. Heat transfer occurs by radiation and heat transfer with a heat trans-

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